COMPARISON OF FAST DECOUPLED LOAD FLOW METHODS AND ITS APPLICATION TO STATIC STATE ESTIMATION

A Thesis Submitted

in Partial Fulfilment of the Requirements

for the degree of

MASTER OF TECHNOLOGY

by

Lt GURUNANDAN GANGOLLI

to the

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

Feb, 1994

CERTIFICATE

Certified that the work entitled 'COMPARISON OF FAST DECOUPLED LOAD FLOW METHODS AND ITS APPLICATION TO STATIC STATE ESTIMATION' being submitted by Lieutenant GANGOLLI GURUNANDAN Roll No 9211508, in partial fulfillment of the award of degree of Master of Technology, has been carried out under our supervision To the best of our knowledge, this thesis has not been submitted elsewhere for the award of a degree

Feb 1994

(Dr P K Kalra)
Associate Professor

(Dr S C Srivastava)
Associate Professor

Department of Electrical Engineering
Indian Institute of Technology
Kanpur - 208016

ACKNOWLEDGEMENT

I am greatly indebted to Dr S C Srivastava and Dr P K Kalra for suggesting this topic, for their valuable guidance and constant encouragement, without which I could not have succeeded in completing this work To work with them has been a lesson in sincerity, dedication and logical thinking It is, therefore, with a deep sense of gratitude that I acknowledge my gratefulness to them

I am highly thankful to all faculty members of I I T Kanpur, who have taught me various courses during my academic program

My sincere thanks to the INDIAN NAVY for giving me the opportunity to undergo this M Tech course

The financial support provided by Council of Scientific and Industrial Research (CSIR), New Delhi for carrying out the present work under project No CSIR/EE/9261 (grant no 8(52)/92-EMR-II)is gratefully acknowledge

Gangolli Gurunandan

TABLE OF CONTENTS

LIST OF TABLES

LIST OF PRINCIPLE SYMBOLS

n = number of buses in the system

δ_i V_i = voltage angle, magnitude at bus i

e_i , f_i = real and imaginary components of the voltage

Y_{ij} = $G_{ij} + jB_{ij}$ = 'ij th element of bus admittance matrix

$P_i + jQ_i$ = net real and reactive power injected at bus i

$p_{ij} + jq_{ij}$ = real and reactive line power flow from ith bus to jth bus

The other symbols in the text are defined as when they are introduced

ABSTRACT

The energy control center is the nodal agency which controls the power system network The load flow solution program is run to determine the power flow in the network The input to this algorithm is the measured data telemetered from field and may get corrupted due to various reasons Hence, the state estimator program is used to estimate the state of the network Thus, the load flow and state estimation are two subproblems of power system network

In this work, the performance of various load flow and state estimator algorithms are critically compared in well and ill-conditioned cases The standard fast decoupled load flow in polar coordinates using Stott's as well as Amerongen s assumptions are compared with the FDLF methods in rectangular and two new algorithms in hybrid coordinates

The literature survey shows that orthogonalisation by Givens rotation has been applied to the state estimation using normal equations Fast decoupled state estimator by normal equations method using the same assumptions of Stott as well as Amerongen is compared with the normal equations using orthogonalisation

A comparative study of all the FDLF and the various state estimator methods has been done in this thesis with reference to computation time and number of iterations for both normal and ill conditioned cases

CHAPTER 1

INTRODUCTION

1 1 GENERAL

The electrical power system is undergoing continuous expansion in size and complexity Power system networks are being provided with computer aided Energy Management system for its effective monitoring and control It is essential that the operating state of the system be continuously monitored, so that this knowledge can be be used for operation and control of the system Energy control centers receive measured data from field through remote terminal units and communication link Determination of system state in absence of any measurement error can be done by load flow or power flow studies whereas it can be performed by state estimator in presence of random error in the measurement

Since the operator's decision for planning, operation and control are based on the knowledge about system operating conditions and many other advanced functions are performed based on the knowledge of current system state, the two subproblems viz, the load flow and state estimation are of vital importance The present thesis addresses to the two problems subproblems and explores few fast methods suitable for their on line applications

Load flow study is a steady state solution of a power system problem which provides information about network power flow and voltage profile Load flow studies on a digital computer essentially involve the solution of non-linear algebraic power

flow equations It is the most frequently carried out study for taking various on-line as well as off-line decisions

State estimation is a software, run on-line at energy control centers and is a process of assigning values to the un-known system state variables using imperfect measurements The state estimator requires the measurements to be redundant and the process of estimating the system states is usually based on a statistical criteria that estimates the true value of the state variables to minimizes or maximize the selected criteria A commonly used and familiar criteria is that of minimizing the sum of the squares of the differences between the estimated and measured values of a function

In the power system, the state variables are the voltage magnitudes and phase angles at the system buses The inputs to the estimator are selected imperfect power system measurements of voltage magnitudes, real power, reactive power or ampere-flow quantities The state estimation can smooth out small random errors in measurements, detect and identity gross measurement errors and fill-in meter readings that have failed due to communication breakdown

The desirable features of these two subproblems are -

i) High speed for real time application

ii) Low and compact storage

iii) Reliability for well behaved and ill-conditioned situations in the networks

1 2 STATE OF THE ART AND MOTIVATION

In the past four decades, a number of research works have been reported on load flow and state estimation problems It is not possible to review all the work reported in these two areas Hence only a representative survey of few important works and those relevent to the present thesis are presented below

1 2 1 LOAD FLOW

The history of load flow analysis dates back to the fifties when method, now known as Gauss-Seidel [1] was used for solving the load flow equations Although this method was very simple and requires very little memory storage, it requires large computational time and presents convergence problems for large as well as ill-conditioned systems

The Newton-Raphson method represented a breakthrough [3] in load flow situation The Newton Raphson load flow (NRLF) method involves the linearization of the set of load flow equations at any iteration point using only the first order term in Taylor's series expansion Several versions of NR method have been developed and the popular ones are based on power mismatches The power mismatches versions can be formulated using either the polar [3] or rectangular co-ordinates [2] for voltages The power mismatch version in polar co-ordinates has advantage over rectangular version in that it requires lesser memory space as the number of variables are reduced

The power mismatch version in polar co-ordinates developed by Tinney and Hart [3] utilizes the sparsity exploitation of optimal ordering of buses and has been widely used by the

utilities This method usually converges to a fairly accurate solution in two to five iterations for most of the systems independent of their size and type Though NRLF method has proved to be highly reliable, the storage and time requirements were considerably high

Decoupled versions of NR method [5,6] were proposed in view of reducing memory and computational time In all these methods, the weak coupling between the active power - voltage magnitude and reactive power - voltage phase angles were exploited Amongst various decoupled versions, fast decoupled load flow (FDLF) method developed by Stott and Alsac [6] is the most popular with utilities The method is in polar co-ordinates and utilizes certain justifiable network assumptions apart from the P-V and Q-δ decoupling to obtain constant Jacobian submatrices The final model of FDLF is developed employing certain additional assumptions in computing elements of the submatrices [B'] and [B''] The FDLF model developed is quite fast and reliable for a wide spectrum of systems

Over the period from 1975 till date, a number of modifications have been proposed to Stott's fast decoupled load flow to improve its speed, convergence characteristics and memory requirements In addition to this, a number of new but similar decoupled versions have been proposed which easily lend them-selves to comparison with Stott's FDLF

Nanda, et al [11] have in their paper brought out clearly the relative importance of some of the assumptions made by Stott and Alsac in their FDLF Their paper states that the omission of

shunt reactances and off-nominal in phase transformer taps in [B] matrix has little effect on convergence of the FDLF and the omission of series resistances, however, plays a vital role in reducing the number of iterations for certain systems

Van Amerongen [18] has studied the effect of neglecting the line resistances and has come up with four schemes wherein the line resistances are neglected or included in either or both the [B'] and [B''] matrices The test results show that for higher R/X ratios the scheme where the line resistances are only neglected in [B '] matrix perform better than as suggested by Stott and Alsac He also advised the strict (1d-1V) iteration scheme to avoid cycling behavior The paper also suggests that cycling can be avoided even with the standard iteration scheme suggested by Stott and Alsac, by using different tolerances for the active and reactive subproblems while carrying out iterations

Rubbi [21] has theoretically shown the differences between the Stott's and Amerongen's FDLF Keyhani [15] has come up with four modified versions of the FDLF in order to reduce the memory requirements for very large systems The algorithm is claimed to require ~3% less memory as compared to the FDLF However, it has a slightly slower rate of convergence than the FDLF

An improved version of the FDLF by Mark Enns [8] presents a new algorithm based on a slight modification of the FDLF This eliminates the calculation of trigonometric functions altogether and yet converges to an exact solution The convergence property is same as for the FDLF and the savings in multiplications is

about 1/4th for each iteration Thus computation time is reduced

Behnam and Guliani [16] have described a load flow method at least 50% faster than Stott and Alsac's FDLF The model used by the authors combines the nodal iterative model (Gauss-Seidal technique) and the FDLF by Stott and Alsac An important and useful feature of this algorithm is that this model does not use [B] matrix and therefore, in contingency studies, the [B''] matrix needs to be refactorised for accommodating the contingencies, this method gives a faster solution than the standard FDLF

The fast decoupled load flow of Stott and Alsac is based on the assumption that the R/X ratios are small for all the branches This condition constitutes a limitation in finding load flow solution for those which have branches having relatively high resistances or where the overall R/X ratio is not small This limitation has elicited many research works in this area

Van Amerongen [18] has come up with one solution as stated earlier Dy Liacco et al [7] have suggested the series compensation method to overcome the problem, but could not compete over the speed Similarly, Monticelli et al [10] have come up with shunt compensation method, which is more reliable than the series compensation method Rajicic and Bose [17] have presented a new modification to the FDLF, which they found, consistently provided better convergence Their modification is based on tests conduct-ed by the authors on systems with high R/X ratios However, their method converges to the solution only for R/X ratio below three Wang et al [22] too have developed

similar technique on further modifying the algorithm proposed by Anjan Bose et al [15]

From the literature survey it is found that the FDLF method based on Stott and Alsac s assumptions [6] are still most popular compared to the other methods and is being used for various on-line and off-line studies The General purpose FDLF method suggested by Amerongen [18] offers advantages in solving systems with high R/X ratio of lines and is also being adopted by utilities Few models suggested in literature do not consider Q-limits of generators

Although much research efforts have been expended in developing fast decoupled load flow models in polar coordinates, the development of its counterpart in rectangular coordinates has not attracted much attention

The main advantage of the rectangular version over the polar version is that the former does not involve trigonometric functions and thus leads to a considerable saving in time spent on the computation of these If decoupling can be realized effectively, then the rectangular FDLF versions would be as competitive as the polar FDLF versions

Raju et al [14] have developed the first order decoupled algorithm whose model is based on all the assumptions made by Stott Their algorithm is in rectangular co-ordinates but the scope of their algorithm is limited to well-behaved systems

Babic [13] too has come up with decoupled load flow in rectangular co-ordinates, which however is not reliable for ill-conditioned systems L Srivastava et al [20] have

formulated three different fast decoupled algorithms in rectangular co-ordinates, one of which exhibits better convergence and is superior to the Stott's FDLF in certain ill-conditioned cases in terms of number of iterations but takes more CPU time Also the methods did not consider Q-limit of generators

Since the power flow equations in rectangular co-ordinates involve only quadratic terms, the complete Taylor's series expansion will be limited to only second order terms, unlike the polar co-ordinates Hence the second order load flow in rectangular co-ordinates, in effect should be more accurate than the polar or first order load flow in rectangular co-ordinates Some if the decoupled versions of second order load flow in rectangular coordinates were suggested by Nanda et al [12] and Cory et al [19], but is not found to be computationally superior to the FDLF in terms of CPU time

The main advantage of the rectangular version over the polar version is that the former does not involve any trigonometric functions and thus leads to a considerable saving in time spent on the computation of these If decoupling can be realized effectively, then the rectangular FDLF versions would be as competitive as the polar FDLF versions

Hence the motivation was to develop two FDLF models in rectangular coordinates using Stott s as well Amerongen's assumptions and considering generator Q-limits In addition, two hybrid algorithms where in the two decoupled equations are in two different coordinates have been developed The performance

of all methods have been compared with the standard FDLF [6,18] in polar coordinates

1 2 2 STATIC STATE ESTIMATION

The static state estimation (SSE) considers the time invariant measurements and static model of the power system One of the earliest methods of state estimation algorithm developed by Schweppe et at [23] is the Weighted Least Squares (WLS) method The WLS algorithm works well and converges fast to a solution point with the systems whose information matrix is well conditioned When the information matrix is numerically ill-conditioned, the WLS algorithm does not perform satisfactorily and manifests this problem in the poor of slow convergence

The fast decoupled static state estimator developed by Horisberger et al [24] utilizes the decoupled P-d and Q-V equations and solves the active and reactive equations iteratively using constant simplified sub-matrices of the information matrix The algorithm yields the exact solution and it requires less computer storage However it could not detect and identify bad data

Garcia [25] introduced fast decoupled state estimation using decoupled detection and bad data identification Bad data is eliminated by pseudo measurement generation and avoids gain matrix retriangulation

Rao et al [26] described an improved decoupled static state estimator based on minimization of WLS residuals No approximation is made in the mismatch functions, the final

solution is as accurate as provided by using complete information matrix The estimator requires considerably less storage and solution time but more iterations They have shown that the rate of convergence of the estimator is significantly improved by adaptively adjusting the step-size during iterations

Monticelli et al [33] developed fast decoupled state estimator in a new frame of reference Decoupling was not seen as zeroing coupling submatrices in the problem Jacobian, but as a two step procedure to solve full Newton equations, without major approximations In the primal algorithm instead of neglecting resistances from [B'] matrices, resistance is neglected from [B''] matrix Whereas dual algorithm is similar to standard fast decoupled state estimator

Srinivasan et al [31] developed three new and improved methods for static state estimation problem, out of which two are based on first order model and the third one is based on second order model Among the three methods the modified Fast Decoupled State Estimator is the fastest algorithm Though the other two methods are not as fast as the modified FDS estimator, they are more reliable than the FDS estimator, i e , well suited for ill-conditioned system

Irving et al [25] introduced power system state estimation using linear programming Linear programming has the combined advantages of noise filtering and bad data elimination and was implemented using the Simplex method El-Keib et al [37] came up with a formulation which improved the solution time

significantly The disadvantages of the LP estimator are that, it does not exploit the problem structure and it fails to reject the bad data due to presence of leveraged points

Kotigua et al [30] used a new method for obtaining an estimate of the state of a power system using Weighted Least Absolute Value (WLAV) WLAV estimator simultaneously detects and rejects bad data while obtaining an accurate estimate of the state Celik et al [35] developed a robust state estimator using transformations, which remains insensitive to bad measurements even when associated with leveraged points These transformations represent a change of co-ordinates in the state-space WLAV estimators lose their robustness in the presence of leveraged points The transformation of Jacobian matrix [H] is to eliminate the leveraged points associated with the original set of measurements WLAV estimator's draw back is its poor computational efficiency for large systems

Soliman et al [36] described a new technique for constrained power system state estimation by considering equality constraints This is based on WLAV state estimation procedure

Rao et al [28] described the Levenberg-Marquardt algorithm to determine the static state estimation for ill-conditioned power systems The diagonal elements of the information matrix of WLS estimator are suitably increased to accelerate the convergence during the iterative solution when the linear model is found to be too approximate

Van Cutsen et al [29] used a hierarchical concept to solve the static state estimation problem for large scale

composite power systems In this algorithm solution is obtained by performing a two level calculation In the lower level, a conventional state estimator is carried out simultaneously for all sub-systems The co-ordination of these local estimations is realized in the upper level, thus it receives and treats only a small number of variables

Labudda et al [38] used fuzzy multi-objective approach to power system state estimation The fuzzy LP estimator offers a number of advantages over the standard LP/WLAV approach, the most noticeable of which is superior robustness in the presence of leveraged bad data

EarsHolten et al [32] compared different methods for state estimation such as Normal equations method, Orthogonal transformation method, Hybrid method, Normal equations with constraints and Hachtel's augmented matrix method

The normal equations method of solving the static state estimation does not permit the calculation of the power system state variables when new measurements are obtained after the base case has been solved The equations technique requires a complete rerun of the solution algorithm with regard to the new measurements

However, the orthogonal transformation method permits the recalculation of base case state variables with respect to the new measurements Narasimhan Vempati et al [34] have developed the transformation using the Givens rotations They have suggested three enhancements, to improve the speed of orthogonalisation for power system state estimation, and make

it comparable with that of the normal equations method

From the literature survey, it appears that the FDSE is being popularly used by utilities Amongst the recently developed methods, the orthogonal transformation method [34] shows promise in achieving the solution much faster than other methods However, the model suggested in the paper was based on NR method Hence the motivation was to extend the orthogonal method using Fast Decoupled equations using both Amerongen's and Stott's assumptions and compare its performance with FDSE based on normal equations

1 3 THESIS ORGANIZATION

The work reported in this thesis has been organized into four chapters

The present chapter - 1 introduces the problem of load flow studies, state estimation and sets the motivation behind the present work

Chapter - 2 is devoted to the comparison of different Fast Decoupled Load Flow models Two algorithms for FDLF in rectangular coordinates and the two using hybrid model have been developed in this chapter Potential of all these methods have been tested on IEEE 14 Bus and 30 Bus test systems for both well-behaved and ill-conditioned cases

Chapter - 3 deals with the Fast Decoupled Power System State Estimator Two different Models of static state estimator has been proposed using orthogonal transformations based on Givens rotations and its performance has been compared with the FDSE using normal equations method The tests have been carried on IEEE 14 Bus test system

Finally, the Chapter - 4 concludes the specific findings in this thesis along with suggestions for future scope of work

CHAPTER II

COMPARISON OF DIFFERENT FDLF METHODS

2 1 INTRODUCTION

The objective of load flow study is to determine the voltage magnitude and angle at each bus, real and reactive power flow in each line and line losses of the power system network for specified bus loading conditions The Newton Raphson's method has proved to be a powerful method for solving the load flow equations but is not used for on-line applications as it requires more memory and computational time

Fast Decoupled Load Flow (FDLF) method suggested by Stott and Alsac [6] overcomes these drawbacks of Newton Raphson method and is probably the most popular method being used by utilities

This method utilises certain physical properties of power system networks such as P-δ and Q-V decoupling, considers assumptions such as lines have small R/X ratio, voltage angle difference at two buses being small In addition the model utilises certain additional assumptions to make the Jacobian matrices [B'] and [B] constant for which no theoritical proof is given by Stott and Alsac [6] Several other models considering these additional assumptions in different combination were suggested and the one become popular is a General purpose FDLF due to Amerongen et al [18], which was subsequently proved by Monticelli et al [ ]

These models were developed in polar coordinates and use sensitivity method to handle Q limits of generators However, their counterparts in the rectangular coordinates considering Q

limits have not been tried at Hence, in this chapter the FDLF models based on Stott's and Amerongen's assumptions have been developed in rectangular coordinates and tested for IEEE 14 and 30 bus systems Two hybrid models FDLF models using both polar and rectangular version of P-δ and Q-V equations have also been tried and their performance compared with the above FDLF methods in polar and rectangular coordinates

2 2 FDLF METHODS IN POLAR COORDINATES

In a practical power system, the following assumptions hold good

i) $(\delta_i - \delta_j)$ is small, thus $COS(\delta_i - \delta_j) \cong 1$

ii) $G_{ij}\ SIN(\delta_i - \delta_j) \ll B_{ij}$

iii) $Q_i \ll B_{ii} * V_i^2$

Consider a power network system to have a total n+1 buses with the (n+1)th bus as the slack bus, (1-m) being P-V type and m PQ type buses The following equations in polar coordinates are solved to get the unknown variables i e the voltage magnitudes and phase angles

$$\left[\frac{\Delta P}{V}\right] = \left[B'\right]\left[\Delta\delta\right] \qquad (2\ 1)$$

$$\left[\frac{\Delta Q}{V}\right] = \left[B''\right]\left[\Delta V\right] \qquad (2\ 2)$$

In the above eqns [B'] and [B''] matrices are constant, real and sparse matrices and need to be triangulated/inverted only once at the beginning of the iteration

Stott and Alsac [6] took the following additional assumptions to arrive at the above mentioned equations

i) Omitting from [B''] the representation of those network elements that affects MVAR flow i e, shunt reactances and off nominal in-phase transformer taps

ii) Omitting from [B'], the angle shifting effects of phase shifters

iii) Neglecting series resistances in calculating the elements of [B']

The above equations form the fast decoupled load flow model [6] on polar coordinates Each iteration cycle comprises of one solution for [$\Delta\delta$] to update [δ] and then one solution for [ΔV] to update [V] and is termed as (1δ,1V) scheme The iterative procedure is repeated till the real and reactive power mismatches become less than a pre-assigned tolerance value

Stott and Alsac's model of FDLF however could not perform well in case the power system network was ill-conditioned, i e , with high R/X ratio Van Amerongen [18] has formulated a general purpose FDLF algorithm in which he has neglected the series line resistance from the [B''] matrix and not from [B'] matrix Amerongen's model of FDLF was able to solve the load flow problem even in ill-conditioned cases To compare the various FDLF models critically for their performance in well and ill conditioned cases the Stott's model and Amerongen's model are termed as FDLF-I and FDLF-II respectively

2 3 FDLF METHODS IN RECTANGULAR COORDINATES

Fast decoupled load flow method [6,18] in polar coordinates, FDLP, discussed earlier is found to be very popular method because of their less memory and CPU time requirement The formulation of a load flow method in rectangular coordinates offers faster solution as compared to its formulation in polar coordinates,

mainly because of the reasons that it does not involve additional computational efforts in solving trigonometric functions With this in view, development of fast decoupled load flow methods in rectangular coordinates (FDLR) have been tried out

FDLF model in rectangular coordinates can be developed using the similar approach as in polar coordinates The Voltage at any bus in rectangular coordinates can be considered as

$$\bar{V}_i = e_i + jf_i \tag{2 3}$$

Assuming that $e \cong 1\ 0$ p u and $Q_{ii} \ll B_{ii}e_i^2$ the load flow jacobians are found to be nothing but that derived from negated susceptance part of the Y_{BUS} matrix Taking additional assumptions as done in the standard FDLF in polar coordinates, the FDLF model in rectangular coordinates are

$$\left[\Delta P\right] = \left[B'\right]\left[\Delta f\right] \tag{2 4}$$

$$\left[\Delta Q\right] = \left[B''\right]\left[\Delta f\right] \tag{2 5}$$

To determine the voltage magnitude of the P-V type buses, the fast decoupled load flow method in rectangular equations has in addition to the eqns (2 4) and (2 5), one more equation which is

$$e^2 = (V^{sp})^2 - f^2 \tag{2 6}$$

To consider the Q limits of the P-V buses, two new models in rectangular coordinates using the sensitivity method has been proposed in this thesis The two model of FDLF in rectangular coordinates taking the Stott's and Amerongen s assumptions are termed as FDLF-III and FDLF-IV respectively

2 4 PROPOSED FAST DECOUPLED METHODS

As has been stated earlier, the fast decoupled method in

polar coordinates is less faster than the formulation in rectangular coordinates because of the additional computational efforts in solving the trigonometric functions However, the number of iterations needed for convergence in rectangular coordinates is found to be greater than that in the polar coordinates Hence, it was with a view to combine the advantages of both the coordinate method that two new formulations in hybrid form have been proposed and has been critically examined for their performance in well and ill-conditioned systems

2 4 1 PROPOSED HYBRID FDLF METHOD 1

In this proposed method, termed FDLF-V the real power mismatch equation is in polar coordinates and the reactive power mismatch equation is in rectangular coordinates The two equations are as follow

$$\left[\frac{\Delta P}{V}\right] = \left[B'\right]\left[\Delta\delta\right] \qquad (2\ 6)$$

$$\left[\Delta Q\right] = \left[B''\right]\left[\Delta f\right] \qquad (2\ 7)$$

A flat voltage (1 0 + j 0) for P-Q buses and V^{spec} for slack and P-V buses is initially started with Since the eqn (2 6) is in polar coordinates, the real power is calculated using bus voltages in polar coordinates and the real power mismatch is determined After the first half iteration is completed, the incremental bus phase angle ($\Delta\delta$) value is used to update the complex bus voltage

The complex bus voltage in polar coordinates is converted to rectangular coordinates and the reactive power is calculated Subsequently, the reactive mismatch is determined and the second half iteration is carried out to obtain (Δe) This is continued till convergence Major computational steps for the proposed model

of FDLF is summarised as following

1) Read system data including line data, loads at P-Q buses, generator data etc

2) Form Y_{BUS} matrix, [B'] and [B''] matrices and P-V buses sensitivity values

3) Initialize the bus voltage to (1.0+j0.0)p.u. for all P-Q buses, V^{spec} for slack and P-V buses

4) Set iteration count K = 0

5) Calculate [ΔP] for all buses except slack bus with bus voltage in polar coordinates

6) Test for convergence of bus real powers (max ΔP ≤ ε) If converged, go to step 10, else go to step 7

7) Advance iteration count by 0.5 (K = K + 1)

8) Solve for Δδ using eqn (2.6)

9) Update δ by Δδ for P-Q and P-V buses

10) Convert the complex bus voltage in polar coordinates to rectangular coordinates

11) Calculate ΔQ for all buses except slack bus and check for convergence If converged goto step 16 else go to step 13

12) For P-V buses, check for violation of Q limits, if so determine Δe by using the sensitivity eqn
$\Delta e = \frac{\Delta Q}{e} * S_i$ where S_i is the sensitivity of the ith P-V bus

13) Advance the iteration count by 0.5 (K = K + 1) and compute Δe using equation (2.7)

14) Update e by Δe for all P-Q buses

15) Convert voltages in polar coordinates and go to step 5

16) Check for convergence of eqn (2.6) If not converged go to step 5

2 4 1 PROPOSED FDLF METHOD II

In this proposed method (termed FDLF-VI) the real power mismatch equation is in rectangular coordinates and the reactive power mismatch equation is in polar coordinates The two mismatch equations are

$$\left[\Delta P\right] = \left[B'\right]\left[\Delta f\right] \tag{2 9}$$

$$\left[\frac{\Delta Q}{V}\right] = \left[B\right]\left[\Delta V\right] \tag{2 10}$$

A flat voltage (1 Ø +jØ) for P-Q buses and V^{spec} for slack and P-V buses is initially started with Since the eqn (2 9) is in rectangular coordinates, the real power is calculated using bus voltages in rectangular coordinates and real power mismatch is determined The bus voltage is updated by Δf after the first iteration

The bus voltage in rectangular coordinates is converted to polar coordinates and reactive power mismatch is calculated The second half of the iteration determines the incremental voltage magnitude ΔV

The computational steps for the model FDLF-VI are similar as discussed in the earliar proposed method

2 6 HANDLING OF PV BUSES

The generator bus which has the largest real power capacity is usually made the reference bus or slack bus, and the other generator buses are termed as the P-V buses In the load flow solution algorithm, the quantities specified for the P-V buses are the voltage magnitude and the real power generation and load The reactive power limits of the generator are known In the

process of calculation of the load flow solution, if the calculated reactive power violates the specified limits for any P-V bus, then two techniques are employed to overcome this problem

i) Bus switching method

ii) Sensitivity method

The Bus switching method is the technique wherein in each iteration the calculated reactive power for the P-V bus which violates the limit (either the maximum or minimum value) is fixed to its corresponding limiting value and the P-V bus is treated as a P-Q bus during that iteration and the change in the voltage magnitude is determined

With the bus switching scheme, if during an iteration a P-V bus violates the Q limit, and is converted to P-Q bus, then a new row and a new column corresponding to that bus is added to the [B''] matrix in FDLF and vice versa Since [B''] matrix has changed, it has to be retriangulated/reinverted This will increase the load flow solution time drastically and thus deteriorates the attractive feature of FDLF

In view of the above, a sensitivity method is used in which the sensitivity of the P-V buses us determined During the load flow solution if the calculated reactive power of a P-V bus violates the specified Q limits, then the incremental voltage magnitude is determined as a product of the reactive power mismatch and the sensitivity to update its specified value instead of converting it to P-Q bus The change in specified voltage is calculated for a P-V bus i as

$$\Delta V = S_i \frac{\Delta Q_i}{V_i}$$

where S_i is sensitivity term derived from diagonal element of inverse of another matrix identical to [B'] but assembled with P-V buses also included This sensitivity matrix is formed only once at the beginning of load flow solution and utilised for Q-limit check in each iteration Thus the sensitivity method of load flow is much faster

2 7 SYSTEM STUDIES

Various fast decoupled load flow models considering both Stott's and Van Amerongen's assumptions as discussed in previous sections (FDLF-I to FDLF-VI) have been tested by solving the load flows for the following power system examples on HP 9000 computer system

1 IEEE 14 bus system

2 IEEE 30 bus system

The system data is given in Appendix 1 & 2 A convergence criterion of 0 0001 p u has been taken using a power base of 100 MVA The results have been obtained for both well behaved and ill-conditioned cases Ill conditioning in the system have been created by increasing R/X ratio of all the lines simultaneously which has been achieved by raising the series resistance of all the lines by a common factor 'α' simultaneously from their base values

Results of unadjusted load flow solutions for the two test systems are given in Tables 1 & 2 which compares the performance of differend load flow models in terms of no of iterations taken by each method for both well behaved and ill conditioned system

The sensitivity method have been used to handle Q-limit in all the above models and the results of adjusted load flow

solutions comparing no of iterations taken by the six versions of FDLF for only IEEE 14 bus system is given in Table 3

Approximate CPU time taken by each iteration of FDLF in polar and FDLF in rectangular is 0 08 and 0 06 sec respectively

2 8 Conclusions

From the results given in tables (2 1 to 2 3) we can conclude that

1 For well behaved system, the number of iteration taken by the standard FDLF taking Stott and Alsac's assumptions is the least for base case for both the 14 and 30 bus unadjusted and 14 bus adjusted cases

2 For ill conditioned cases, the General purpose FDLF algorithm by Amerongen in polar coordinates takes the least iterations followed by the FDSE V (in hybrid coordinates)

3 The FDSL IV using Amerongen s assumptions in rectangular coordinates is the worst for both the 14 and 30 bus systems

From this it is concluded that FDLF II using Amerongen's assumptions in polar coordinates is the best method

TABLE 2 1

Comparison of various FDLF models for unadjusted load flow solution (IEEE 14 bus)

Value of α to increase R/X ratio	No of Iterations					
	FDLF I	FDLF II	FDLF III	FDLF IV	FDLF V	FDLF VI
1 0	4 0	4 5	4 5	5 5	6 0	5 5
1 5	4 5	5 5	6 5	div	6 5	6 5
2 0	6 5	5 5	9 5	div	6 5	8 5
2 5	8 5	5 5	11 5	div	7 5	9 5
3 0	11 0	6 0	37 5	div	6 5	11 5
3 5	14 0	6 0	36 0	div	9 0	div

TABLE 2 1

<u>Comparison of various FDLF models for unadjusted load flow solution (IEEE 30 bus)</u>

Value of α to increase R/X ratio	No of Iterations					
	FDLF I	FDLF II	FDLF III	FDLF IV	FDLF V	FDLF VI
1 0	3 5	4 5	5 5	5 5	5 0	4 5
1 5	5 5	5 0	8 5	7 5	5 5	7 0
2 0	7 5	5 0	12 5	div	5 5	10 5
2 5	11 0	5 0	19 5	div	6 0	17 0
3 0	15 0	6 0	div	div	6 5	div
3 5	21 0	6 5	div	div	10 0	div

TABLE 2 3

Comparison of various FDLF models for adjusted load flow solution (IEEE 14 bus)

Value of α to increase R/X ratio	No of iterations					
	FDLF I	FDLF II	FDLF III	FDLF IV	FDLF V	FDLF VI
1 0	5 0	5 5	5 5	5 5	5 5	6 0
1 5	5 5	5 5	6 5	div	6 5	6 5
2 0	6 5	5 5	7 5	div	8 5	6 5
2 5	8 5	7 5	11 5	div	9 5	7 5
3 0	9 5	6 5	15 5	div	11 5	6 5
3 5	10 5	11 5	18 5	div	div	11 5

CHAPTER 3

NEW FAST DECOUPLED STATE ESTIMATOR MODELS

3 1 INTRODUCTION

The problem of monitoring the power flows and voltages on a transmission system is very important for maintaining the system security By checking each measured value against its limit, one can identify where problems exist in the network and can take corrective actions to relieve overloaded lines or out-of-limit voltages

These, and many other advanced functions such as optimal power despatch, unit commitment, automatic generation control etc are performed at the energy control centers and requires a reliable system data base The system data acquired from field, may be corrupted when received at energy control centers due to inherent inaccuracies of measuring devices, transducers, A/D converters and the telemetry link If the measurement errors are small, they may go undetected and can cause misinterpretation

A state estimator can smooth out small random errors in meter readings, detect and identify gross measurement errors, and fill in the missing readings that might have not been acquired due to communication failure One of the state estimation methods most popularly being used by utilities at energy control centers, is the Fast Decoupled state estimator (FDSE) [ ] based on the Weighted Least Square (WLS) technique The FDSE is quite fast as it uses constant information matrices in each iteration However, even FDSE method, state estimator

program may take few minutes to estimate the state of a moderate size system whereas the data scan time varies between one to ten seconds The research has been proceeding towards developing newer and faster algorithms in order to achieve this goal but still remains a challenge to researchers in this area

Narasimhan Vempati et al [34] suggested a new algorithm using orthogonalization of the normal equations method to solve the static state estimation problem Their algorithm is more robust than the normal equations method and is claimed to be extremely fast in updating the state for small change in the measurement set However, they have applied Newton Raphson (NR) equations for the state estimation The fast decoupled model is known to be superior in speed and time over the NR method for solving the load flow problem The motivation behind the present works was to develop a model based on orthogonalization of fast decoupled equations

A comparison of the various load flow methods shown in the previous chapter indicates that the General purpose load flow algorithm suggested by Van Amerongen [18] performs better than the Stott's FDLF [6] specially in solving ill-conditioned cases The motivation was to develop FDSE model using Amerongen's assumptions and considering normal as well as orthogonalised equations

Thus, in this chapter four different models of FDSE has been developed, two based on normal equations and the two using orthogonalization, utilizing Stott's as well as Amerongen's load flow models

3 2 WEIGHTED LEAST SQUARE METHOD IN POWER SYSTEM STATE ESTIMATION

The three criteria on which the static state estimation are generally developed are the maximum likelihood criterion, the weighted least squares criterion and the minimum variance criterion Since the weighted least squares (WLS) criterion is of relevance to this work it is described briefly in this section The objective used in the WLS method is to minimize the sum of the squares of the weighted deviations of the estimated measurements from the actual measurements

The system data received at the energy control center may be corrupted due to various reasons as stated earlier The difference between the true and the measured quantity can be termed as the measurement error and its distribution is random We can assume that the probability density distribution function of such a random error is a normal (Gaussian) distribution

Let Z^{mea} be the measured value of the system data of measurements, Z^{true} be the true value and η be the random measurement of the error The measurement value can be expressed as

$$Z^{mea} = Z^{true} + \eta \qquad (2\ 1)$$

If σ is the standard deviation and σ^2 is the variance of the measurement error, the objective function can be written as

$$\min_{x} J(X) = \sum_{i=1}^{N_m} \frac{\left(Z^{mea} - Z^{true}\right)^2}{\sigma^2} \qquad (2\ 2)$$

The measured quantity could be a function of the state variable where h is the functional relationship The measurements

can be modelled in an unified manner as

$$Z = h(X) + \eta \tag{2.3}$$

where Z is a (mx1) measurement vector, X is (nX1) system state, h(X) is the (mx1) vector of non-linear measurement functions and η is a (mx1) vector of measurement errors.

Assuming sufficient redundancy in the measurement i.e., m>n, the best estimate of the state is obtained by minimizing the weighted least squares objective function

$$\min J(X) = \left[Z - h(X) \right]^{T} R^{-1} \left[Z - h(X) \right] \tag{2.4}$$

Rewriting the objective function in an incremental form to accommodate the non-linearity of the problem, the objective can be written as

$$\min J(\Delta X) = \left[\Delta Z - H(\Delta X) \right]^{T} R^{-1} \left[\Delta Z - H(\Delta X) \right] \tag{2.5}$$

where Z is the (mx1) measurement mismatch vector, H is a (mxn) measurement Jacobian, X is the (nx1) correction vector to the system state and R is the (mxm) diagonal covariance matrix. Setting the gradient of the objective function to zero for minimization, the normal equations method obtains the estimate by iteratively solving

$$\left[H^{T} R^{-1} H \right] \Delta X = H^{T} R^{-1} \Delta Z \tag{2.6}$$

where $H^{T} R^{-1} H^{T}$ is known as information matrix.

3.3 FAST DECOUPLED STATE ESTIMATOR

In fast decoupled state estimator the measurement vector Z is partition into active and reactive parts i.e.

$$Z = \begin{bmatrix} Z_p \\ Z_q \end{bmatrix} = \begin{bmatrix} f(V,\delta) \\ g(V,\delta) \end{bmatrix} \simeq h\,(V,\delta) \tag{2.7}$$

where Z and Z are the sets of active and reactive power injections power flow measurement, respectively

With the above definitions we obtain the Jacobian

$$H(\delta,V) = \begin{bmatrix} \dfrac{\partial P}{\partial \delta} & \dfrac{\partial P}{\partial V} \\ \dfrac{\partial Q}{\partial \delta} & \dfrac{\partial Q}{\partial V} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \qquad (2\ 8)$$

For a practical system, the elements of H_{11} and H_{22} are generally much larger than those of H_{12} and H_{21} respectively Hence the latter Jacobian submatrices are neglected

Thus, the eqn (2 8) takes the form

$$H(\delta,V) = \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \qquad (2\ 9)$$

The information matrix will be

$$I = \begin{bmatrix} H_{11}^{t}\, R_{p}\, H_{11} & 0 \\ 0 & H_{22}^{t}\, R_{q} H_{22} \end{bmatrix} \qquad (2\ 10)$$

where R_p and R_q are the real and reactive power measurement covariance matrices respectively Denoting H_{11} by H_p and H_q by H_{22}, we get two decoupled sets of equations for state estimator as follows

$$\begin{bmatrix} H_{p}^{t}\, R_{p}\, H_{p} \end{bmatrix} \Delta\delta = H_{P}^{t}\; R_{P}\, \Delta P \qquad (2\ 11)$$

$$\left[H_q^t R_q H_q \right] \delta V = H_q^t R_q \Delta Q \qquad (2.12)$$

The active and reactive power injections, line flows can be expressed

$$P_i = V_i \left[\sum_{i=1}^{n} V_j G_{ij} \cos(\delta_i - \delta_j) + \sum_{i=1}^{n} V_j B_{ij} \sin(\delta_i - \delta_j) \right] \qquad (2.13)$$

$$Q_i = V_i \left[\sum_{i=1}^{n} V_j G_{ij} \sin(\delta_i - \delta_j) - \sum_{i=1}^{n} V_j B_{ij} \cos(\delta_i - \delta_j) \right] \qquad (2.14)$$

$$P_{ij} = \left[V_i^2 - V_i V_j \cos(\delta_i - \delta_j) \right] g_{ij} - V_i V_j b_{ij} \sin(\delta_i - \delta_j) \qquad (2.15)$$

$$Q_{ij} = -\left[V_i^2 - V_i V_j \cos(\delta_i - \delta_j) \right] b_{ij} - V_i V_j g_{ij} \sin(\delta_i - \delta_j) - V_i^2 y_{shij} \qquad (2.16)$$

where $G_{ij}+jB_{ij}$ is the ijth element of the Y_{bus} and $g_{ij}+j\ b_{ij}$ is the series admittance of the line connected between bus i and j

The elements of the Jacobian corresponding to real power measurement (H) using fast decoupled assumptions [6] are

$$H_p = \begin{bmatrix} \dfrac{\partial P_i}{\partial \delta} \\ \dfrac{\partial P_{ij}}{\partial \delta} \end{bmatrix} \qquad (2.17)$$

The elements of [B'] and[B'] will be different if the above FDSE model is developed based on Amerongen's assumptions as compared to the one developed based on Stott's assumptions These two models have been referred as FDSE-2 and FDSE-1 respectively In FDSE-1 model while forming [B'] the series resistances of lines, shunts and transformer in phase tappings are omitted, while forming [B'] phase shifters are omitted In FDSE -2 model while forming [B'] the series resistances of lines are not omitted, while forming [B] the series resistances of the lines and phase shifters are omitted

Applying these assumptions the fast decoupled state estimator model is based to solved to following equations in each iteration

$$\Delta\delta = \left[H_p^T R_p^{-1} H_p \right]^{-1} H_p^T R_p^{-1} \Delta Z_p \qquad (2\ 18)$$

$$\Delta V = \left[H_q^T R_q^{-1} H_q \right] H_q^T R_q^{-1} \Delta Z_q \qquad (2\ 19)$$

3 4 STATIC STATE ESTIMATOR BASED ON ORTHOGONAL TRANSFORMATION

The linearized objective function eqn (2 5)can be rewritten in terms of square root of inverse of covariance matrix (R) as

$$J(\Delta X) = \left[R^{-1/2}\Delta Z - R^{-1/2}H\ \Delta X \right]^T \left[R^{-1/2}\Delta Z - R^{-1/2}H\ \Delta X \right] \qquad (2\ 20)$$

Define A to be an orthogonal (mxm) matrix such that $A^T A = I$ using orthogonal transformation based on Given's rotation (Appendix 3 1),

$$A\ R^{-1/2}H = \begin{bmatrix} D^{1/2}\ U \\ \emptyset \end{bmatrix} \qquad (2\ 21)$$

where D is a diagonal (nxn) matrix and U is a unit upper triangular (nxn) matrix On account of orthogonality of A, the objective function can be rewritten as

$$J(\Delta X) = \left[A\ R^{-1/2}\Delta Z - A\ R^{-1/2}H\ \Delta X \right]^T \left[A\ R^{-1/2}\Delta Z - A\ R^{-1/2}H\ \Delta X \right] \qquad (2\ 22)$$

The orthogonal transformation (Appendix 3 1) can be applied to the measurement vector also,

$$A\ R^{-1\ 2}\Delta Z = \begin{bmatrix} D^{-1\ 2}\Delta Z_1 \\ D^{-1\ 2}\Delta Z_2 \end{bmatrix} \qquad (2\ 23)$$

Minimization of the objective function leads to

$$U \Delta X = \Delta Z_1 \qquad (2.24)$$

Back substitution of Z on U provides X and hence the new state vector (X) If the Jacobian is not considered to be constant then orthogonalization is of both Jacobians and measurement mismatch is required in each iteration till convergence is achieved In order to save CPU time Narasimhan Vempati et el [34] have reline arized Jacobian after every four iteration

The equation (2 24) can also be directly used to find the change in system state for a small incremental change in measurement set

3 5 PROPOSED NEW FAST DECOUPLED STATE ESTIMATOR

In the proposed method, the Jacobian submatrices have been made constant by applying fast decoupled assumptions and thus the computational time required for repeatedly orthogonalizing the Jacobian in each iteration can be saved The constant real and reactive Jacobians are transformed only once to form the orthogonal matrices A and A , which are used in successive iterations This model has been developed using Stott's as well as Amerongen's assumptions for [B] and [B''] matrices and have been referred as FDSE-3 and FDSE-4models respectively

The major computational steps for the proposed new fast decoupled state estimators are listed below

1 Read system data including the telemetered measurements
2 Form Y_{bus} matrix, the constant jacobian submatrices and the covariance matrices
3 Set iteration count K = 0
4 Rotate rows of H_p into U_p one by one and form the corresponding orthogonal matrix A
5 Rotate rows of H_q into U_q one by one form the corresponding orthogonal matrix A_q

6 Compute Z_1 to obtain ΔZ_{1p} and ΔZ_{2p} by applying the rotation operations of step 4

7 Perform back substitution on ΔZ_{1p} on U_p to obtain $\Delta\delta$

8 Update bus voltage angles δ by $\Delta\delta$

9 Compute Z_q Transform Z_q to obtain Z_{1q} and Z_{2q} by applying the rotation operations of step 9

10 Advance iteration count by K = K + 1

11 Perform back substitution on Z_{1q} on U_{1q} to obtain ΔV

12 Update V by ΔV

13 If convergence is not achieved, go to step - 6

3 6 SYSTEM STUDIES

The four state estimator models FDSE-I, FDSE-II, FDSE-III and FDSE-IV discussed in this chapter have been tested on the IEEE 14 and 30 Bus test system on HP 9000 computer system A convergence criteria of 0 0001 p u on state variables (voltages) has been taken using a power base of 100 MVA A base case load flow solution was obtained as the measurement set of data required for testing the state estimation programs Errors were introduced in both the test systems in the real and reactive over injections bus no 4 & 6 as well as in the real and reactive power flows of lines 8 & 15 An error of 3% (increase) was considered from the base case values in the real power quantities and 2% (decrease) was considered in the reactive power quantities Rest of the measurements set were not altered for running the state estimation program The weight matrix elements for the perfect measurements were considered as 1000, whereas for the imperfect measurements were taken as 100 The result obtained from the four methods are presented in table 3 1 and 3 2 for 14 bus system and table 3 3 and 3 4 for 30 bus

system Table 3 1 and 3 3 compares the number of iterations and tables 3 2 and 3 4 compare the state variables Incremental state estimation was carried out using all the four models by introducing a 5% increase in base case line flow for real power line No 10 Using the previously calculated base case state variables (Voltage magnitudes and angles), the state estimation program was run again to determine the incremental change in state variables The no of iterations required for calculating the incremental change has been given in Table 3 5

The CPU times required per iteration using FDSE based on normal equations are 0 04sec and 0 047sec for 14 and 30 bus system respectively and using orthogonal methods are and sec respectively

3 5 Conclusion

From the results presented in table 3 1 to 3 5, one can conclude the following

i) For the full state estimation run the number of iterations required by FDSE-2 and FDSE-4 (both based on Amerongen's assumptions) take the minimum iterations compared to other two methods The orthogonal method using Stott's assumptions (FDSE-3) takes two iteration less as compared to the normal equations (FDSE-1) method for 14 bus system However, they take same number of iterations for 30 bus system

ii) For an incremental change in the measurement set,the FDSE-3 performs worst in case of 14 bus system, whereas all other methods require identical number of iterations for both 14 and 30 bus test systems

iii) A comparison of CPU time reveals that orthogonal method, as formulated in the present work, takes slightly more time per iteration compared to the normal equation method, which can be

improved by incorporating the enhancements suggested in ref [34] and the sparsity techniques

In view of the above observations it is recommended to use FDSE-4 model

TABLE 3 1

Comparison of various FDSE methods for IEEE 14 bus test system

FDSE Methods	No of iterations for base case
FDSE I	9
FDSE II	7
FDSE III	8
FDSE IV	6

TABLE 3-2

COMPARISON OF STATE VARIABLES (IEEE 14 BUS TEST SYSTEM)

BUS= 1	V(mea)=1 0600	V(estm)=1 0600	D(mea)= 0000	D(estm)= 0000	V(esmt)=1 0599999	d(esmt)= 0000000
BUS= 2	V(mea)=1 0450	V(estm)=1 0454	D(mea)=- 0783	D(estm)=- 0772	V(esmt)=1 0523953	d(esmt)=- 0801697
BUS= 3	V(mea)=1 0700	V(estm)=1 0620	D(mea)=- 2030	D(estm)=- 1731	V(esmt)=1 0539857	d(esmt)=- 2041321
BUS= 4	V(mea)= 0100	V(estm)=1 0114	D(mea)=- 2083	D(estm)=- 2057	V(esmt)=1 0177447	d(esmt)=- 2085571
BUS= 5	V(mea)=1 0900	V(estm)=1 0815	D(mea)=- 2092	D(estm)=- 1799	V(esmt)=1 0756699	d(esmt)=- 2098083
BUS= 6	V(mea)=1 0624	V(estm)=1 0543	D(mea)=- 2092	D(estm)=- 1801	V(esmt)=1 0477439	d(esmt)=- 2098389
BUS= 7	V(mea)=1 0554	V(estm)=1 0490	D(mea)=- 2332	D(estm)=- 2036	V(esmt)=1 0404383	d(esmt)=- 2346802
BUS= 8	V(mea)=1 0276	V(estm)=1 0301	D(mea)=- 1371	D(estm)=- 1226	V(esmt)=1 0284317	d(esmt)=- 1364746
BUS= 9	V(mea)=1 0216	V(estm)=1 0240	D(mea)=- 1631	D(estm)=- 1478	V(esmt)=1 0219423	d(esmt)=- 1621399
BUS=10	V(mea)=1 0503	V(estm)=1 0449	D(mea)=- 2329	D(estm)=- 2032	V(esmt)=1 0340775	d(esmt)=- 2340698
BUS=11	V(mea)=1 0563	V(estm)=1 0486	D(mea)=- 2204	D(estm)=- 1907	V(esmt)=1 0386437	d(esmt)=- 2210083
BUS=12	V(mea)=1 0544	V(estm)=1 0497	D(mea)=- 2190	D(estm)=- 1812	V(esmt)=1 0350374	d(esmt)=- 2193298
BUS=13	V(mea)=1 0504	V(estm)=1 0462	D(mea)=- 2219	D(estm)=- 1849	V(esmt)=1 0319774	d(esmt)=- 2228088
BUS=14	V(mea)=1 0351	V(estm)=1 0305	D(mea)=- 2458	D(estm)=- 2105	V(esmt)=1 0176181	d(esmt)=- 2473755

USING STOTTS ASSUMPTIONS (NORMAL EQUATIONS METHOD) (ORTHOGONAL METHOD)

BUS= 1	V(mea)=1 0600	V(estm)=1 0600	D(mea)= 0000	D(estm)= 0000	V(esmt)=1 0599999	d(esmt)= 0000000
BUS= 2	V(mea)=1 0450	V(estm)=1 0455	D(mea)=- 0783	D(estm)=- 0764	V(esmt)=1 0523752	d(esmt)=- 0802004
BUS= 3	V(mea)=1 0700	V(estm)=1 0644	D(mea)=- 2030	D(estm)=- 1767	V(esmt)=1 0537720	d(esmt)=- 2040733
BUS= 4	V(mea)=1 0100	V(estm)=1 0115	D(mea)=- 2083	D(estm)=- 2018	V(esmt)=1 0182142	d(esmt)=- 2087955
BUS= 5	V(mea)=1 0900	V(estm)=1 0825	D(mea)=- 2092	D(estm)=- 1780	V(esmt)=1 0753452	d(esmt)=- 2098117
BUS= 6	V(mea)=1 0624	V(estm)=1 0553	D(mea)=- 2092	D(estm)=- 1798	V(esmt)=1 0474104	d(esmt)=- 2098118
BUS= 7	V(mea)=1 0554	V(estm)=1 0499	D(mea)=- 2332	D(estm)=- 2115	V(esmt)=1 0400568	d(esmt)=- 2348911
BUS= 8	V(mea)=1 0276	V(estm)=1 0322	D(mea)=- 1371	D(estm)=- 1142	V(esmt)=1 0275732	d(esmt)=- 1360409
BUS= 9	V(mea)=1 0216	V(estm)=1 0259	D(mea)=- 1631	D(estm)=- 1392	V(esmt)=1 0214685	d(esmt)=- 1618942
BUS=10	V(mea)=1 0503	V(estm)=1 0473	D(mea)=- 2329	D(estm)=- 2070	V(esmt)=1 0337688	d(esmt)=- 2341736
BUS=11	V(mea)=1 0563	V(estm)=1 0523	D(mea)=- 2204	D(estm)=- 1917	V(esmt)=1 0384427	d(esmt)=- 2210199
BUS=12	V(mea)=1 0544	V(estm)=1 0550	D(mea)=- 2190	D(estm)=- 1787	V(esmt)=1 0349077	d(esmt)=- 2191341
BUS=13	V(mea)=1 0504	V(estm)=1 0509	D(mea)=- 2219	D(estm)=- 1837	V(esmt)=1 0318270	d(esmt)=- 2225422
BUS=14	V(mea)=1 0351	V(estm)=1 0360	D(mea)=- 2458	D(estm)=- 2083	V(esmt)=1 0173538	d(esmt)=- 2471333

USING AMERONGEN'S ASSUMPTIONS (NORMAL EQUATIONS METHOD) (ORTHOGONAL METHOD)

TABLE 3 3

Comparison of various FDSE methods for IEEE 30 bus test system

FDSE Methods	No of iterations for base case
FDSE I	6
FDSE II	6
FDSE III	6
FDSE IV	6

TABLE 3.4

COMPARISON OF STATE VARIABLES BETWEEN STOTT'S AND AMERONGEN'S METHOD BY ORTHOGONAL TECHNIQUE

(30 BUS TEST SYSTEM)

	STOTT'S METHOD		AMERONGEN'S METHOD	
BUS= 1	V(estm)=1 0600	D(estm)= 0000	V(estm)=1 0600	D(estm)= 0000
BUS= 2	V(estm)=1 0457	D(estm)=- 0752	V(estm)=1 0456	D(estm)=- 0751
BUS= 3	V(estm)=1 0159	D(estm)=- 1404	V(estm)=1 0156	D(estm)=- 1404
BUS= 4	V(estm)=1 0640	D(estm)=- 2021	V(estm)=1 0639	D(estm)=- 2025
BUS= 5	V(estm)=1 0121	D(estm)=- 2156	V(estm)=1 0120	D(estm)=- 2155
BUS= 6	V(estm)=1 0537	D(estm)=- 2202	V(estm)-1 0586	D(estm)=- 2195
BUS= 7	V(estm)=1 0059	D(estm)=- 1842	V(estm)=1 0058	D(estm)=- 1843
BUS- 8	V(estm)=1 0245	D(estm)=- 1063	V(estm)-1 0243	D(estm)=- 1064
BUS= 9	V(estm)=1 0417	D(estm)=- 2011	V(estm)=1 0414	D(estm)=- 2008
BUS=10	V(estm)=1 0366	D(estm)=- 2288	V(estm)=1 0363	D(estm)=- 2285
BUS=11	V(estm)=1 0158	D(estm)=- 1275	V(estm)=1 0155	D(estm)=- 1278
BUS=12	V(estm)=1 0474	D(estm)=- 2201	V(estm)=1 0473	D(estm)=- 2191
BUS=13	V(estm)=1 0145	D(estm)=- 1458	V(estm)=1 0143	D(estm)=- 1460
BUS=14	V(estm)=1 0311	D(estm)=- 2341	V(estm)=1 0307	D(estm)=- 2333
BUS=15	V(estm)=1 0273	D(estm)=- 2343	V(estm)-1 0269	D(estm)=- 2336
BUS=16	V(estm)=1 0345	D(estm)=- 2285	V(estm)=1 0341	D(estm)=- 2281
BUS=17	V(estm)=1 0301	D(estm)=- 2325	V(estm)=1 0297	D(estm)=- 2322
BUS=19	V(estm)=1 0156	D(estm)=- 2442	V(estm)=1 0152	D(estm)=- 2439
BUS=19	V(estm)=1 0132	D(estm)=- 2470	V(estm)=1 0127	D(estm)=- 2469
BUS=20	V(estm)=1 0186	D(estm)=- 2438	V(estm)=1 0180	D(estm)=- 2436
BUS=21	V(estm)=1 0237	D(estm)=- 2351	V(estm)=1 0232	D(estm)=- 2350
BUS=22	V(estm)=1 0244	D(estm)=- 2344	V(estm)=1 0239	D(estm)=- 2342
BUS=23	V(estm)=1 0193	D(estm)=- 2358	V(estm)-1 0188	D(estm)=- 2351
BUS=24	V(estm)=1 0188	D(estm)=- 2317	V(estm)=1 0183	D(estm)=- 2306
BUS=25	V(estm)=1 0287	D(estm)=- 2016	V(estm)=1 0282	D(estm)=- 1995
BUS=26	V(estm)=1 0227	D(estm)=- 2012	V(estm)=1 0218	D(estm)=- 1981
BUS=27	V(estm)=1 0352	D(estm)=- 1874	V(estm)=1 0349	D(estm)=- 1854
BUS=28	V(estm)=1 0141	D(estm)=- 1490	V(estm)=1 0140	D(estm)=- 1491
BUS=29	V(estm)=1 0261	D(estm)=- 1925	V(estm)=1 0258	D(estm)=- 1898
BUS=30	V(estm)=1 0181	D(estm)=- 2028	V(estm)=1 0181	D(estm)=- 1993

TABLE 3 3

Comparison of various FDSE methods for incremental change

FDSE Methods	No of iterations	
	14 bus	30 bus
FDSE I	4	2
FDSE II	2	2
FDSE III	4	2
FDSE IV	2	2

CHAPTER IV

CONCLUSION

The work carried out in this thesis was aimed at comparing different fast decoupled load flow methods in polar and rectangular coordinates as well as exploring new solution techniques for power system load flow studies The standard FDLF by Stott and Alsac does not behave well in ill conditioned cases, whereas the General purpose FDLF by Amerongen can handle even if the network system is ill conditioned The potential of existing FDLF methods using Stott's as well as Amerongen s assumptions have been compared with the new algorithms on IEEE 14 and 30 bus systems for both well and ill conditioned cases

A comparative study of the performance of the various FDLF techniques indicated the best method which could be used to develop a new fast decoupled state estimator

Narasihman Vempati et al [34] have applied the orthogonalisation technique to the state estimator However their work has been in Newton method They have claimed that the orthogonalisation makes the algorithm more robust in ill conditioned cases In this thesis the orthogonalisation by Givens rotation has been applied to the fast decoupled state estimator using Stott's as well as Amerongen's assumptions Hence, an attempt has been made to explore the possibility of combining both the advantages

From the comparison of the test results of the various FDLF and FDSE methods presented in Tables 2 1 to 3 5, it is concluded that

1 The general purpose fast decoupled load flow by Van Amerongen is found to take the least iterations in ill conditioned systems, however the standard FDLF by Stott and Alsac takes the least number of iterations for well behaved systems

2 Between the two algorithms using Amerongen's assumptions, the one in polar coordinates (FDLF II) takes lesser number of iterations than FDLF VI

3 The time taken per iteration by the FDLF methods in polar coordinates is slightly greater than in rectangular coordinates

4 For the base case study of fast decoupled state estimator, the method using orthogonalisation by Givens rotation taking Amerongen s assumptions is the best for both the base case and incremental estimation The FDSE method by normal equations using Stott's assumptions is the worst in IEEE 14 bus test systems However all the methods perform equally well for the 30 bus test system

5 The time taken by the state estimator using normal equations is lesser than using the orthogonalisation method

The orthogonalisation procedure is computationally intensive and hence the time taken by this method is more than the normal equations method Vempati et al [34] have however described the enhancement to the Givens rotation by which the number of

multiplications is reduced They have also exploited the sparsity of the Jacobian by applying various sparsity techniques which has not been done in this thesis

The bad data measurement is one of the foremost problems to be tackled in the study of state estimation Narasihman Vempati et al [34] have claimed that by applying the orthogonalisation principle to the state estimator problem, the identification of bad data and its removal is easier Hence, the present work may be extended in future to incorporate this important feature in fast decoupled state estimator programs, which could enhance its utility

REFERENCES

LOAD FLOW

[1] Ward J B and Hale H W , Digital Computer Solution of Power Flow Problem AIEE Trans (PAS), Vol 75, June 1956, pp 394-404

[2] Vannes J E and Griffin J H , Elimination Methods for Load Flow studies , AIEE Trans (PAS), Vol 80, June 1961, pp 299-304

[3] Tinney W F and Hart C E , Power Flow Solutionh of Newton s Method", IEEE Trans (PAS), Vol 86, Nov 1967, pp 583-588

[4] Tinney W F and Walker J W , Direct Solutions of Sparse Network Equations by Optimally Ordered Triangular Factorisation , Proc IEEE, Vol 55, Nov 1967, pp 1801=1809

[5] Stott B , Decoupled Newton Load Flow , IEEE Trans , Vol PAS-91, Sept/Oct 1972, pp 1955-1959

[6] Stott B and Alsac O , Fast Decoupled Load Flow , IEEE Trans , Vol PAS-83, May/June 1974, pp 859-869

[7] Dy Liacco and Rama Rao K A , comments on Theoritical Study on Convergence of FDLF , by Wu F F , IEEE Trans , Vol PAS-96, No 1, Jan 1977, pp 268-278

[8] Mark Enns, An Improved Version of FDLF , Proc IEEE , Vol 65, Feb 1977, pp 278-279

[9] Abc S , Hamada N , Isono A , Okuda K , Load Flow Convergence in the Viscinity of a Voltage Stability Limit , IEEE Trans PAS-97, No 6, Nov/Dec 1978, pp 1983-1993

[10] Deckman S , Pizzolinte A , Monticelli A , Stott B , and Alsac O , Appendix to Numerical Testing of Power System Load Flow Equivalents , IEEE Trans Vol PAS-99, No 6, Nov/Dec 1980, pp 2292-2300

[11] Nanda J , Kothari D P , and Srivastava S C , Some Important Observations on the FDLF Algorithm , Proc IEEE, Vol 75, May 1987, pp 732-733

[12] Nagendra Rao P S , Rao K S P , and Nanda J , An Exact Load Flow Method Including Second Order terms in Rectangular Coordinates , IEEE Trans Vol PAS-101, No 9, Sep 1982, pp 3261-3268

[13] Babic B S , Decoupled Load Flow with variables in Rectangular Form , Proc IEE, Part C, Vol 130, 1983, pp 1865-1868

[14] Singh S P , Raju G S , Subbarao V s , A Constant Matrix Flow Method , Proc of Third National Power System Conference, REC Warangal, India, 1984, pp 1101-1107

[15] Keyhani A , Study of FDLF Algorithm with Substantially Reduced Memory Requirements , EPSR , Vol 9, No 1, June 1985, pp 1-9

[16] Behnam K and Guliani, FDLT Hybrid Model , IEEE Trans on PS , Vol 3, No 2, May 1988,pp 734-742

[17] Rajcic D and Anjan Bose, A Modification to Fast Decoupled Power Flows for Networks for High R/X Ratios , IEEE Trans on PS, Vol 3,No 2, May 1988, pp 743-746

[18] Robert Van Amerongen, A General Purpose Version of FDLF , IEEE Trans , on PS, Vol 4, No 2, May 1989, pp 760-770

[19] Cory B J , Ekwue A O , and Johnson R B I , Comparative Analysis of the Convergence Characteristics of 2nd Order Load Flow Methods , Electrical Power and Energy Systems , Vol 12, No 1, Oct 1990, pp 251-256

[20] Srivastava L , Srivastava S C , and Singh L P , Fast

Decoupled Methods in Rectangular Coordinates , Electrical Machinery and Power System (EMPS), Vol 13, No 3 June 1991, pp 160 169

[21] Hubbi W , Effects of neglecting resistances in XB and BX Load Flow Methods , Proc IEE Part C, Vol 138, No 5, Sept 1991, pp 152 456

[22] Wang L , Xiang N , Wang S , Zhang B , and Huang M , General Purpose Fast Decoupled Load Flow , Proc IEE , Part C, Vol 137, No 1, March 1992, pp 1-7

STATE ESTIMATION

[23] Schweppe F C and Wildes J , Power System Static State Estimation , Part I Exact Model , IEEE Trans Vol PAS-89, No 1 Jan 1970 pp 120 130

[24] Horisberger H P , Richard J C and Rossier C , A Fast Decoupled static state estimator for electric power systems , IEEE Trans Vol PAS-95, No 1, Jan/Feb 1976, pp 208-215

[25] Irving M R , Owen R C and Sterling M J H , Power System state estimation using Linear Programming , Proc , IEE Vol 125, No 9, Sept 1978, pp 879-885

[26] Rao N D and Tripathy S C , A variable step size decoupled state estimator , IEEE Trans , Vol PAS-98, NO 2, March/April 1979, pp 436-443

[27] Garcia A and Monticelli A and Abren P , Fast decoupled State estimation and bad data processing , IEEE Trans Vol PAS 98, No 5, Sept/Oct 79, pp 1645 1652

[29] Rao N D and Tripathy S C , Power system static state

estimation by the Levenberg Marquardy algorithm , IEEE Trans Vol PAS 99, No 6, Nov/Dec 1980 pp 695 702

[29] Van Cutsem , Howard J I and Ribbens-pavalle M , A two level static state estimator for electric power systems , IEEE Trans Vol PAS 100, No 8, Aug 1981, pp 3722-3732

[30] Willy Kotigua W and Vidyasagar M , Bad data rejection properties of Weighted least absolute value techniques applied to static state estimation , IEEE Trans Vol PAS 101, No 4,April 1982, pp 844 853

[31] Srinivasan N , Rao K S P and Indulkar C S , Some new algorithm for state estimation in power system , IEEE Trans Vol PAS 103, No 5, May 1984, pp 982 987

[32] Lars Holten, Anders Gjelsvik, Sverre Aam, Felix Wu and Wen Hsiung Lin E , Comparison of diferent methods for state estimator , IEEE Trans on power systems, Vol 3, No 3, Aug 1988, pp 830 836

[33] Monticelli A and Garcia A , A fast decoupled state estimator , IEEE Trans on power systems, Vol 5, No 2, May 1990, pp 556-564

[34] Narasihman Vempati, Ilya Slutsker W and Tinney W F Enhancements to Givens Rotations for power system state estimation , IEEE Trans on power systems, Vol 6, No 2, May 1991, pp 556-564

[35] Mehmet Celik K and Ali Abus, Use of scaling in WLAS estimation of power systems states , IEEE Trans on power systems Vol 7, No 1, Feb 1992, pp 106-113

[36] Soliman S A , Christensen G S and Rouhi A H , Power system

APPENDIX A

Consider the following two row vectors

$$\bar{U} = \left[\emptyset \, , \, , \, , \emptyset \, , U_i \, , \, , \, , U_j \, , \, , \, , U_{r+1} \right]$$

$$\bar{X} = \left[\emptyset \, , \, , \, , \emptyset \, , X_i \, , \, , \, , X_j \, , \, , \, , X_{r+1} \right]$$

A plane rotation between $\bar{U}$ and $\bar{X}$ is defined such that the ith element of $\bar{X}$ is annihilated After the rotation the row vectors take the form

$$\bar{U}' = \left[\emptyset \, , \, , \, , \emptyset \, , U'_i \, , \, , \, , U'_j \, , \, , \, , U'_{n+1} \right]$$

$$\bar{X}' = \left[\emptyset \, , \, , \, , \emptyset \, , \emptyset , \, , \, , X_k \, , \, , \, , X_{n+1} \right]$$

the two vectors $\bar{U}$ and $\bar{X}$ are defined as

$$\begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{bmatrix} \bar{U} \\ \bar{X} \end{bmatrix} = \begin{bmatrix} \bar{U}' \\ \bar{X}' \end{bmatrix}$$

where $C^2 + S^2 = 1$

The scalars C and S are determined from the requirement that $X_i = \emptyset$ and are given by

$$C = \frac{U_i}{\sqrt{U_i^2 + X_i^2}}$$

$$S = \frac{X_i}{\sqrt{U_i^2 + X_i^2}}$$

The above is known as orthogonal transformation using Givens rotation

APPENDIX - B

DATA FOR IEEE-14 BUS TEST SYSTEM (AT 100 MVA BASE)

B-1 Bus Data

Bus No	Generation MW	Generation MVAR	Load MW	Load MVAR
1	0 0	0 0	0.0	0 0
2	40.0	0 0	21 7	12 7
3	20 0	0 0	94 2	19 0
4	0 0	0 0	11.2	7 5
5	0.0	0 0	0.0	0 0
6	0 0	0 0	47 8	-3 9
7	0.0	0 0	0.0	0 0
8	0.0	0 0	7.6	1 6
9	0 0	0 0	29.5	16 6
10	0.0	0 0	9 0	5 8
11	0.0	0.0	3.5	1 8
12	0.0	0.0	6.1	1.6
13	0.0	0 0	13.5	5.8
14	0.0	0 0	14.9	5 0

B-2 Line Data

Line No	Buses From	Buses To	Line Impedance R in p u	Line Impedance X in p u	Half Line Charging susceptance (p.u)
1	1	2	0 01938	0 05917	0 0264
2	1	8	0 05403	0.22304	0 0246
3	2	3	0 04699	0 19797	0 0219
4	2	6	0 05811	0.17632	0 0187
5	2	8	0 05695	0 17388	0 0700
6	3	6	0 06701	0 17103	0 0173
7	4	11	0 09498	0 19890	0 0
8	4	12	0 12291	0 25581	0 0
9	4	13	0 06615	0 13027	0 0
10	6	7	0 0	0 20912	0 0
11	6	8	0 01335	0.04211	0 0064
12	6	9	0 0	0.55618	0 0
13	7	5	0 0	0.17615	0 0
14	7	9	0 0	0.11001	0 0
15	8	4	0 0	0.25202	0 0
16	9	10	0 03181	0 08450	0 0
17	9	14	0 12711	0 27038	0 0
18	10	11	0 08205	0 19207	0 0
19	12	13	0 22092	0.19988	0.0
20	13	14	0 17093	0.34802	0.0

B-3 Transformer Data

Transformer	Buses From	Buses To	Tap setting
1	6	7	0 978
2	6	9	0 969
3	8	4	0.932

A-4 Shunt Capacitor Data

Bus No.	Susceptance p u
9	0.190

B-5 Voltage-Controlled Bus Data (P-V buses)

Bus No.	Voltage magnitude p u	Reactive Power Limits Minimum MVAR	Maximum MVAR
2	1.045	- 40 0	50 0
3	1 010	0 0	40.0
4	1.070	- 6 0	24 0
5	090	- 6 0	24 0

Slack bus voltage = 1 06/0^0

APPENDIX - C

DATA FOR IEEE-30 BUS TEST SYSTEM (AT 100 MVA BASE)

C-1 Bus Data

Bus No.	Generation MW	Generation MVAR	Load MW	Load MVAR
1	0 0	0 0	0 0	0.0
2	40.0	0 0	21 7	12 7
3	0 0	0 0	30 0	30 0
4	0 0	0 0	0 0	0 0
5	0 0	0 0	94 2	19 0
6	0.0	0 0	0 0	0 0
7	0 0	0 0	22 8	10 9
8	0 0	0 0	2 4	1 2
9	0 0	0 0	0 0	0 0
10	0 0	0 0	5 8	2 0
11	0 0	0 0	7 6	1 6
12	0 0	0 0	11 2	7 5
13	0 0	0.0	0 0	0 0
14	0 0	0 0	6 2	1.6
15	0.0	0 0	8 2	2 5
16	0.0	0 0	3 5	1 8
17	0 0	0 0	9.0	5.8
18	0 0	0 0	3 2	0.9
19	0.0	0 0	9 5	3.4
20	0.0	0 0	2 2	0.7
21	0.0	0 0	17 5	11.2
22	0.0	0 0	0 0	0 0
23	0.0	0 0	3 2	1.6
24	0.0	0 0	8.7	6 7
25	0.0	0 0	0.0	0.0
26	0 0	0 0	3 5	2 3
27	0.0	0 0	0 0	0 0
28	0.0	0 0	0 0	0.0
29	0.0	0 0	2.4	0.9
30	0.0	0 0	10 6	1.9

C-2 Line Data

Line No.	Buses From	Buses To	Line Impedance R in p.u	Line Impedance X in p.u.	Half Line Charging susceptance (p u)
1	1	2	0 0192	0.0575	0 0264
2	1	8	0 0452	0.1852	0 0204
3	2	11	0 0570	0.1737	0 0184
4	8	11	0.0132	0.0379	0 004
5	2	5	0 0472	0 1983	0 0209
6	2	13	0 0581	0 1763	0 0187
7	11	13	0 0119	0 0414	0 0045
8	5	7	0 0460	0 1160	0 0102
9	13	7	0 0267	0 0820	0 0085
10	13	3	0 0120	0.0420	0 0045
11	13	9	0 0	0.2080	0 0
12	13	10	0 0	0 5560	0 0
13	9	4	0 0	0 2080	0 0
14	9	10	0 0	0 1100	0.0
15	11	12	0 0	0 2560	0 0
16	12	6	0.0	0 1400	0 0
17	12	14	0.1231	0 2559	0 0
18	12	15	0 0662	0 1304	0 0
19	12	16	0.0945	0 1987	0 0
20	14	15	0 2210	0.1997	0 0
21	16	17	0 0824	0.1923	0 0
22	15	18	0 1070	0 2185	0 0
23	18	19	0 0639	0.1292	0 0
24	19	20	0 0340	0.0680	0 0
25	10	20	0 0936	0.2090	0 0
26	10	17	0 0324	0 0845	0.0
27	10	21	0 0348	0 0749	0 0
28	10	22	0 0727	0 1499	0.0
29	21	22	0 0116	0 0236	0.0
30	15	23	0.1000	0 2020	0 0
31	22	24	0 1150	0.1790	0.0
32	23	24	0.1320	0 2700	0 0
33	24	25	0.1885	0.3292	0 0
34	25	26	0 2544	0.3800	0 0
35	25	27	0 1093	0.2087	0 0
36	28	27	0.0	0.3960	0.0
37	27	29	0.2198	0 4153	0.0
38	27	30	0 3202	0.6027	0 0
39	29	30	0 2399	0 4533	0 0
40	3	28	0 0636	0.2000	0 0214
41	13	28	0.0169	0.0599	0.0065

C-3 Transformer Data

Transformer	Buses From	Buses To	Tap setting
1	11	12	0.932
2	13	9	0.978
3	13	10	0.969
4	28	27	0.968

C-4 Shunt Capacitor Data

Bus No.	Susceptance p.u
10	0.190
24	0.043

C-5 Voltage-Controlled Bus Data (P-V buses)

Bus No.	Voltage magnitude p.u.	Reactive Power Limits Minimum MVAR	Reactive Power Limits Maximum MVAR
2	1.045	− 40.0	50.0
3	1.010	− 10.0	40.0
4	1.082	− 6.0	24.0
5	1.010	− 40.0	40.0
6	1.073	− 6.0	24.0

Slack bus voltage = $1.06\angle 0^{\circ}$

Date Slip

This book is to be returned on the date last stamped